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SEQUENTIAL DYNAMIC MODE DECOMPOSITION FOR A FLOW PAST A SPHERE

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Modal decomposition techniques allow better understanding of the phenomena rising in the fluid flow, including the loss of stability and transition to limit-cycle oscillation. They are also crucial enablers of model-based [4], real-time flow control.

The most popular method of decomposition of the signal onto spatial and temporal components is Proper Orthogonal Decomposition [1] and its variant, called method of Snapshots. Known limitations of these methods, especially in the reconstruction of the transitional flow (increasing amplitude of oscillation) and in further application in reduced order modelling, resulted in the intensive research on the decomposition methods. Among the novel techniques like DPOD [5] and BPOD [6], Dynamic Mode Decomposition (DMD) [3, 2] is gaining the most interest.

DMD allows to extract the characteristic coherent structures of the flow, called modes, basing on the assumption that the current state of the flow might be approximated by a linear combination of the previous ones:

$$q(t + \triangle t) \approx e^{\triangle t A} q(t) \tag{1}$$

The linear operator $\tilde{A} = e^{\triangle t A}$ is then replaced by a Companion matrix S, which coefficients $c_0 ... c_n$ are obtained from the solution of the overdetermined system of equations (1). The eigenvectors of matrix S are used to obtain the DMD modes, while the eigenvalues determine modal growth ratios σ and frequencies ω.

In this paper, Sequential Dynamic Mode Decomposition of the flow past a sphere is performed. The input data for numerical experiment is obtained from Direct Numerical Simulation of incompressible Navier-Stokes equation, done by in-house CFD code in the parallel environment. The solver is based on quadratic finite-element formulation and is using MPI routines for the communication between processors and Metis library for domain partitioning.

Simulation is prepared using the sphere of unit diameter placed in a cuboid domain with dimensions 30 x 10 x 10 units. The domain is discretized using 353310 second order tetrahedral elements. The Reynolds number is set to $Re = 400$ and the transient flow from steady state to the limit cycle oscillation at the laminar regime is observed.

Using series of Dynamic Mode Decompositions of the successive sets of snapshots (shifted with respect to the previous ones), it is possible to observe the temporal changes in the vortex street and modal shapes during the development of the vortex street. The sequence of Dynamic Mode Decompositions captures the curves describing parameters like modal frequencies, growth ratios and energetic levels (fig.1), that otherwise are assumed

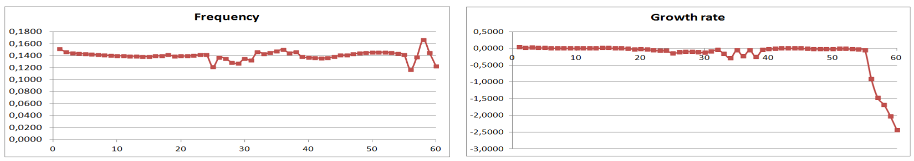

Figure 1: Evolution of the most dominant mode's frequency and growth ratio in sequential DMD analysis

constant due to the use of linear operator. The analysis of these changes allows better understanding of the vortex shedding and modal energy cascade phenomena. The set of modal bases resulting from sequential DMD might be used in the design of parametrized modal basis, that is required to broaden the scope of the reduced-order models of the flow.

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