

## 2D AND 3D GLOBAL STABILITY ANALYSIS BASED ON THE MODAL DECOMPOSITION OF marginally STABLE FLOWS

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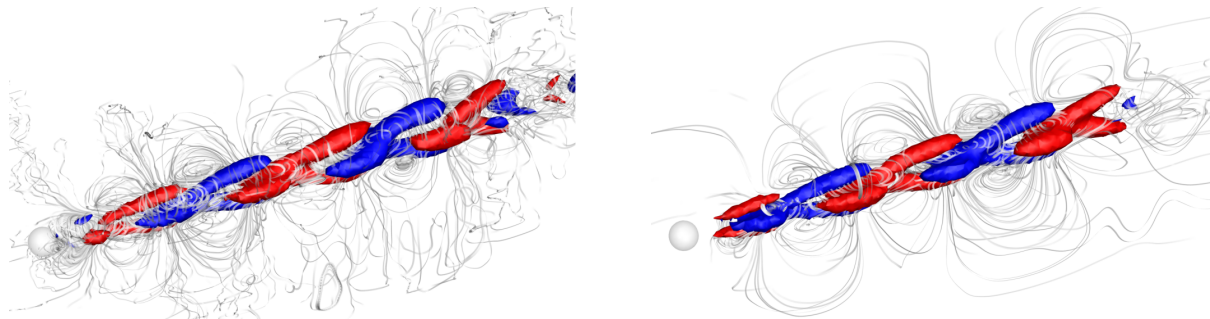
Reduced order models are key enablers of feedback flow control. One of the most popular methods of model order reduction for fluid flows is Galerkin method. It is based on the projection of (approximated) governing equations onto a subspace spanned by mathematical, physical or empirical modes. The right choice of the modal basis has a significant impact on the scope of applicability of the model [6].

In the case of most popular empirical mode bases, resulting from Proper Orthogonal Decomposition [3] of periodic flow undergoing limit cycle oscillations, Galerkin models are unable to capture properly the dynamics of the transition from steady solution to the fully periodic flow (e.g. von Karman vortex street), as well as the influence of the vortex suppression resulting from active flow control. To widen the scope of the model, physical modes, like the ones resulting from global stability analysis [4], have to be incorporated, either as a part of hybrid model [6] or using mode basis parameterization, like continuous mode interpolation [8].

Unfortunately, global stability analysis of general three-dimensional flows is very computationally demanding task. The issues like the solution of large, generalized eigenproblems and the choice of preconditioning making eigenvalues of the real part close to zero the dominant ones result in the very few analysed configurations [5]. These restrictions lead to research on new methods of obtaining physical modes [1, 2].

In this paper, an alternative method of obtaining physical mode basis is presented. The discussed approach is based on Dynamic Mode Decomposition [7] of the snapshots from marginally stable flow, calculated using direct numerical simulation of Navier-Stokes equations. The examples include incompressible, 2D flow past a circular cylinder and a 3D flow past a sphere. The dynamic modes of the flow near fixed-point dynamics (fig. 1)

reproduce the structures of global stability eigenmodes, and the eigenvalues of a Companion matrix resemble the ones of generalized eigenproblem resulting from linearization of Navier-Stokes equations.



**Figure 1:** Real and imaginary parts of the most dominant dynamic modes for the flow past a sphere.

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